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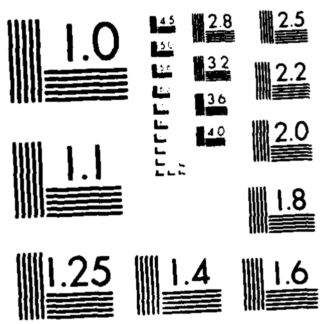

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MEASURING EQUITY IN PUBLIC RISK

by

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Measuring Equity in Public Risk

by

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Abstract

An equitable distribution of risk is often an important criterion in the evaluation of technological and environmental risks. We define the concept of equity precisely and develop measures for ex-post and ex-ante equity. It is shown how these measures along with the information on the number of possible fatalities can be used to rank alternatives that involve risks to human life.

Measuring Equity in Public Risk

An equitable distribution of risk is often an important criterion in the evaluation of technological and environmental risks. Starr and Whipple [1982] state that in risk analysis in national decision making "the central question is with the equity of risk distribution, rather than with comparisons of risk and other costs with benefit." The concept of equity has, however, not been clearly defined in the literature.

Keeney [1980] defined a concept of equity and showed that this concept is in conflict with an attitute to avoid catastrophes. Recently, Broom [1982] observed that Keeney's concept of equity is inappropiate if there is a concern for ex-ante equity. Specifically, he showed that using Keeney's definition of risk equity the sure consequence of individual 1 living and invidual 2 dying will be indifferent to a lottery in which individual 1 lives and individual 2 dies with a .5 chance and individual 2 lives and individual 1 dies with a .5 chance. At a more general level Diamond [1967] had argued that ex-ante equity or the equity of the process violates the sure-thing property of the von-Neumann Morgenstern utility function (substitution of one consequence for another when they are indifferent does not alter the preferences).

To clarify the notion of equity (justice or fairness), we present two precisely defined measures of equity. We show how these measures along with the information on the number of fatalities can be used to evaluate public risks. For clarity of exposition we will deal with a two person society. The distinction is however not biological and two groups such as the residents of earthquake-unsafe and earthquake-safe buildings or the workers in a nuclear plant and the "outsiders" would fall in the same framework. Our results could however be easily generalized to an N-person society. In some instances

such a generalization would require some additional assumptions that should be clear from the context.

The reader should also consult Fishburn [1983] who provides an analysis of compatability among alternative axioms for equity and Keeney and Winkler [1983] who employ a von-Neumann-Morgenstern utility function for evaluating equity in public risk.

1. Notation and Motivation

Consider two individuals 1 and 2 who are exposed to risks of dying due to a specified cause in a specified time period. At the end of the time period one of the four basic consequences will be observed: (1, 1) = both individuals die, (0, 0) = both individuals live, (1, 0) = 1 dies and 2 lives, (0, 1) = 1 lives and 2 dies. At the beginning of the time period we will assume that the probability distribution over the four consequences has been specified. From this probability distribution we can easily derive the marginal probability, denoted m_i, that individual i will be a fatality.

We now consider the following probability distributions over the basic consequences (hereafter called lotteries).

lottery 1	.5	(1, 1)
	.5	(0, 0)
lottery 2	.5	(1, 0)
	.5	(0, 1)
lottery 3	.25	(1, 1)
	.25	(1, 0)
	.25	(0, 1)
	.25	(0, 0)
lottery 4	1	(1, 0)

The expected number of fatalities for each of the above four lotteries is one. However, there may be different evaluations of these, even for a risk neutral decision maker, if there is a concern for equity. Notice that in lotteries 1, 2, and 3 each individual has a .5 chance of dying thus these three lotteries are equitable ex-ante. At the end of the time period lottery 1 produces identical outcomes, lottery 2 produces non-identical outcomes, and lottery 3 produces indentical or non-identical outcomes depending on which of the four possible events does actually occur. Thus, the ex-post equity for the three lotteries differ. We next develop a measure for ex-post equity that considers whether at the end of the time period, when the actual outcomes become known, the distribution of these outcomes is equitable.

2. Ex-Post Equity Index

At the end of the time period one of the four basic consequences (or 2^N in the case of an N-person society) will be realized. The ex-post equity index makes an assumption about the relative equity of these basic consequences and provides a rule for ranking lotteries by the degree of their ex-post equity.

Our expost equity index (Θ) will be based on a binary relation $\succsim_e$, "is more or equally equitable than," defined on a set of lotteries. The asymmetric and symmetric parts of $\succsim_e$ will be denoted $>_e$ ("is more equitable than") and $\sim_e$ ("is equally equitable as"). We also assume that $\succsim_e$ is a weak order (i.e., it is complete and transitive).

We now state two assumptions that allow us to construct the ex-post equity index. To state our first assumption we define a concept of the total number of inferior pairwise comparisons for a basic consequence that is obtained by counting, for each individual, the number of people whose outcomes

are inferior to his and then simply summing all the numbers of inferior comparisons over all individuals. Thus (1, 0, 0, 0) and (1, 1, 1, 0) will each have three inferior pairwise comparisons and (1, 1, 0, 0) will have four inferior pairwise comparisons. For a basic consequence c let us denote n(c) as the number of inferior pairwise comparisons.

Assumption 1.

If $n(c) \leq n(c')$, then $c \gtrsim_e c'$.

By assumption 1, a consequence with a smaller number of inferior pairwise comparisons is judged more equitable. Notice that in equally equitable consequences the variance of the outcomes will be the same as will the maximum number (or percentage) of people with identical outcomes. Thus assumption 1 could be stated in many different, but equivalent, ways to provide the same ranking of the consequences by the desirability of their ex-post equity. We now have a complete ranking in terms of $\gtrsim_e$ for all possible 2^N basic consequences. We denote the most equitable consequences as s and the least equitable consequences as f. For the N = 2 case, $(1, 1) \sim_e (0, 0) \equiv s$, and $(1, 0) \sim_e (0, 1) \equiv f$.

Assumption 2

A non-degenerate lottery ℓ_1 is considered more or equally equitable than a non-degenerate lottery ℓ_2 if and only if $p^1 \geq p^2$, where

$$\ell_1 \equiv \begin{array}{l} \xrightarrow{\;p^1\;} s \\ \xrightarrow[\;1-p^1\;]{} f \end{array}$$

$$\ell_2 \equiv \begin{array}{l} \xrightarrow{\;p2\;} s \\ \xrightarrow[\;1-p^2\;]{} f \end{array}$$

This assumptions says that a more equitable lottery is the one that yields a higher probability of the more equitable outcomes.

We note that for $N \leqq 3$ any lottery can be represented as a two outcome lottery yielding s and f. We define $\theta(s) = 1$ and $\theta(f) = 0$. Thus, the equity ordering is the same as the ordering by the probability of obtaining s.

Theorem 1. If assumptions 1 and 2 are satisfied, then the ex-post equity index of a lottery ℓ, $\theta(\ell)$, is given by $\theta(\ell) = p$, where p is the probability of obtaining s in the lottery ℓ. Further,

$\ell \underset{\sim e}{>} \ell^1$ if and only if $\theta(\ell) \geqq \theta(\ell^1)$.

We now can construct an ex-post equity index for any lottery. For example, $\theta(\ell) = .5$ for lottery 3 in section 1. For an $N > 3$ society, every consequence must first be reduced to an indifferent lottery between s and f and then assumption 2 is used to compute equity index as in Theorem 1. There will at most be $\frac{N}{2} - 1$ (or the next lower integer) θ values different from 0 and 1. For example, in a four person society the ex-post equity index for the consequence (1, 1, 1, 0) is obtained by asking the decision maker to specify p such that he considers this consequence equally equitable to a lottery with a p chance of obtaining the most equitable consequence (1, 1, 1, 1) and a (1 - p) chance of obtaining the least equitable consequence (1, 1, 0, 0). Using theorem 1, θ (1, 1, 1, 0) = p. In fact, θ for any consequence with three fatalities will be p. An example of θ for a lottery yielding y fatalities with a $\Pi(y)$ probability may be

$$\theta = a + b\,\theta', \text{ where}$$

$$\theta' = \sum_{y=0}^{N} ((2y - N)^2/N^2)\,\Pi(y),$$

and a and b > 0 are constants so that $\theta(s) = 1$ and $\theta(f) = 0$.

Suppose the number of fatalities, y, and the ex-post equity θ are the only two concerns in an evaluation. Then, we can asses the two attribute von-Neumann-Morgenstern utility function, $u(y, \theta)$, that can be used to rank lotteries. A simple form of $u(y, \theta)$ results if we make the following assumption.

Assumption 3.

If two lotteries have the same expected utility for the attribute "number of fatalities" and the same ex-post equity index, then these are indifferent.

An immediate consequence of this assumption is that $u(y, \theta)$ is additive.

Theorem 2. Given that assumption 3 holds, the utility function $u(y, \theta)$ is represented by

$$u_Y(y) + \lambda\theta,$$

where λ is a scaling constant.

The shape of the overall utility function $u(y, \theta)$ depends on the shape of the component functions and on the magnitude of the scaling constant. The ex-post equity index is symmetric in y with the highest values at $y = 0$, and $y = N$, and the lowest value at $y = N/2$. One would expect $u_Y(y)$ to be concave, specially when N is large. The overall utility function could be concave or convex.

A problem with this equity concept is that no consideration is given to the ex-ante risks to individuals. For example, the ex-ante risks to each individual in lotteries 1, 2, and 3 are identical (a .5 chance of dying), yet those actions are strictly ordered due to ex-post equity considerations. A somewhat bothersome implication of this development is that lotteries 2 and 4 will be judged indifferent since $y = 1$ and $\theta = 0$ in both cases. Clearly, the

lottery 2 will seem fairer than the lottery 4 to most people. In the next section we deal with this issue and show how the concept of ex-ante equity can be used to alleviate this problem.

3. Ex-ante Equity

At the time the decision about a public risk is made the relevant information is the probability of death that each individual faces. We define marginal probability of death to an individual i as m_i. The ex-ante equity concept deals with the relative distribution of these marginal probabilities. Thus ex-ante equity is represented as $\phi = f(m_1, m_2, \ldots, m_N)$, where f has some desirable properties that are consistent with our intuitive notion of equity. Let us denote $\bar{m}_{ij} \equiv (m_1, m_2, \ldots, m_{i-1}, m_{i+1}, \ldots, m_{j-1}, m_{j+1}, \ldots, m_N)$. If we define the binary relation $\underset{\sim}{>}_{e'}$ on the set of marginal probability distributions for each individual, then we require:

i) if for any i and j, $|m_i - m_j| \leq |m_i' - m_j'|$, then $(m_i, m_j, \bar{m}_{ij}) \underset{\sim}{>}_{e'} (m_i', m_j', \bar{m}_{ij})$,

ii) $(m, m, \ldots, m)_{\sim e'} (m', m', \ldots, m')$ for all $m, m' \in [0,1]$.

In figure 1 these requirements are pictorially depicted for a fixed level of $\bar{m}_{ij}$. The direction of arrow shows the increasing value of ϕ (increasing equity).

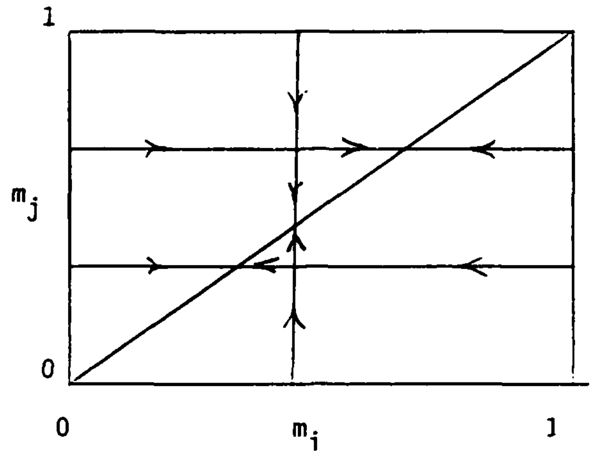

Figure 1

A special form of the function f that can be used to define ϕ is given by the following assumption.

Assumption 4.

$$\phi = -\sum_{i=1}^{N} \left| m_i - \bar{m} \right| ,$$ where $\bar{m}$ is the average probability of death.

Several other measures of ϕ are possible. In general, one may regard the assessment of f as analogous to multiattribute preference assessment except in this case the judgments are made on relative equity.

Now suppose that the number of fatalities, y, and the ex-ante equity, ϕ are the only two concerns in a public risk evaluation. In this case, however, since ϕ is defined over marginal probabilities alone, two identical probability distributions over deaths may not be indifferent. This is because the marginal proabablilities of death may not be equally ex-ante equitable in the two identical probability distributions over deaths. For example, both lotteries 2 and 4 yield 1 death with probability one; however, the former

lottery is strictly preferred if we assume that given the same probability distribution over fatalities a more ex-ante equitable lottery is preferred.

In order to obtain a complete ranking of lotteries we need to estimate ϕ and the expected deaths or the certainty equivalent , $\hat{y}$, for the probability distribution over number of fatalities. To compare lotteries we need to consider the tradeoffs between $\hat{y}$ and ϕ. In general, we need $v(\hat{y}, \phi)$, where v is the two attribute value function (see Keeney and Raiffa [1976, Chapter 3], Dyer and Sarin [1979]). A subtle but significant point that deserves some explanation is that we cannot allow an expectation operation over v. In other words, a vonNeumann-Morgenstern utility function defined over $(\hat{y}, \phi)$ will be invalid. To see this suppose an action a leads to a consequence (1, 0) with a utility value of u and an action b leads to (0, 1) with a utility value of u. Thus the expected utility of a strategy in which actions a or b are chosen with a .5 chance each will also be u. However, the directly computed utility of this strategy will be greater than u -- a clearly inconsistent result. Further, even if one is careful in summarizing all information in terms of marginal probabilities before assessing utilities a problem could arise. To illustrate this denote the component utility function for the attribute "ex-ante equity" which depends on m_1 and m_2 as $u_\phi(m_1, m_2)$. Now, suppose $m_1 + m_2 = 1$; to assess $u_\phi(.6, .4)$, a standard method is to elicit p such that a lottery yielding a p chance at the most equitable outcome (.5, .5) and a (1-p) chance at the least equitable outcome (1, 0) is indifferent to (.6, .4). A logical response for p is .8. Infact, any other response will produce inconsistent results with the requirement that utility should increase with decreasing absolute deviation between m_1 and m_2. This is undoubtedly an undesirable result since the feelings for ex-ante equity will differ in different contexts.

To avoid such paradoxes and unnecessary confusion we will only require a value function over $\hat{y}$ and ϕ. This two attribute value function can be assessed by appropriate methods (e.g. see Keeney and Raiffa [1976]).

One approach to assessing a value function is to convert each lottery into an equivalent lottery with $\phi = 0$ (or some other chosen value). Thus obtain $\bar{\hat{y}}$ such that

$$(\hat{y}, \phi) \sim (\bar{\hat{y}}, 0)$$

The values of $\bar{\hat{y}}$ can then be used to rank order lotteries.

4. Ex-post and Ex-ante equity

We now combine the results in sections 2 and 3 to derive a general risk evaluation model when the number of fatalities, ex-post equity, and ex-ante equity are relevant concerns.

First, for a given lottery k we assess the probability distribution over the number of fatalities denoted $\tilde{y}_k$, the ex-post equity index Θ_k, and the ex-ante equity index ϕ_k. Now, we fix $\Theta_k = \bar{\Theta}$ for all lotteries and seek the certainty equivalent for each lottery k, $(\hat{y}_k, \bar{\Theta}, \phi_k)$ such that

$$u(\hat{y}_k/\bar{\Theta}, \phi_k) = Eu(\tilde{y}_k/\bar{\Theta}, \phi_k).$$

Now, since all lotteries differ only in $\hat{y}$ and ϕ we can assess the value function $v(\hat{y}, \phi)$ as discussed in section 3 to rank order the lotteries.

More specialized evaluation models are possible if we assume additional independence conditions among the three attributes. For example, suppose the two attribute utility function for fatalities and ex-post equity does not depend on the levels of ϕ and is of the additive form as in section 2.

$$v(y, \Theta) = u_Y(y) + \lambda\Theta.$$

Further, suppose that the value function over ϕ and $u(y, \Theta)$ is linear (see Keeney and Raiffa [1976]) and thus we can write it as

$$v(u(y, \Theta), \phi) = K_Y\, u_Y(y) + K_\Theta\, \Theta + K_\phi\, \phi.$$

where K_Y K_Θ, and K_ϕ are nonnegative scaling constants and are normalized so that $K_Y + K_\Theta + K_\phi = 1$. This specialized version of our general model is similar to Keeney and Winkler [1983] who employ a multiattribute utility function instead of a multiattribute value function and define component utility functions over fatalities and marginal probabilities to capture ex-post and ex-ante equity for which we use the specific indices Θ and ϕ.

To illustrate the model let us suppose.

$$u_Y(y) = -\left(\frac{y}{N}\right)^2 ,$$

$$\Theta = \sum_{y=0}^{N} ((2y - N)^2/N^2)\Pi(y)$$

$$\phi = -\sum_{i=1}^{N} \left| m_i - \bar{m} \right| .$$

For the four lotteries in section 1 this model will give

$$v(1) = -.5\, K_Y + K_\Theta$$

$$v(2) = -.25\, K_Y$$

$$v(3) = -.375\, K_Y + .5\, K_\Theta$$

$$v(4) = -.25\, K_Y - K_\phi$$

In all cases lottery 2 is non-inferior to lottery 4 since it is better on ex-ante equity and equal on the other two attributes. Preference orderings for lotteries 1 to 3 depend on the relative values of K_Y and K_Θ. For example, $K_\Theta = K_Y/4$ implies indifference among the three lotteries; $K_\Theta > K_Y/4$ indicates lottery 1 is preferred to lottery 3 which is preferred to lottery 2; and $K_\ni < K_Y/4$ causes a reversal of this ordering.

5. Conclusions

We have provided a measure of ex-post equity and a measure of ex-ante equity that are useful in evaluating public risks. It is shown how these

measures along with the information on expected number of fatalities can be used to rank alternatives that involve risks to human life. This development provides a clear separation of number of fatalities, ex-ante equity, ex-post equity, and trade-offs among these. An advantage of our approach is in clarifying some confusion in the literature that has arisen because of the lack of a precise meaning of the term "equity."

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